NEET | IIT-JEE | FOUNDATION

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JEE MAINS-2019

09-04-2019 Online (Morning)

IMPORTANT INSTRUCTIONS

1. The question paper has three parts: **Chemistry, Mathematics and Physics** Each part has three sections.
2. Section 1 contains 8 questions. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive).
   Marking Scheme: +4 for correct answer and 0 in all other cases.
3. Section 2 contains 10 multiple choice questions with one or more than one correct option.
   Marking Scheme: +4 for correct answer, 0 if not attempted and –2 in all other cases.
4. Section 3 contains 2 “match the following” type questions and you will have to match entries in Column I with the entries in Column II.
   Marking Scheme: for each entry in Column I, +2 for correct answer, 0 if not attempted and – 1 in all other cases.

PART-A-CHEMISTRY

1. Liquid 'M' and liquid 'N' form an ideal solution. The vapour pressures of pure liquids 'M' and 'N' are 450 and 700 mmHg, respectively, at the same temperature. Then correct statement is:

(x_M = Mole fraction of 'M' in solution ;

x_N = Mole fraction of 'N' in solution ;

y_M = Mole fraction of 'M' in vapour phase ;

y_N = Mole fraction of 'N' in vapour phase)

(1*) $\frac{x_M}{x_N} > \frac{y_M}{y_N}$ (2) $\frac{x_M}{x_N} = \frac{y_M}{y_N}$ (3) $\frac{x_M}{x_N} < \frac{y_M}{y_N}$ (4) $(x_M - y_M) < (x_N - y_N)$

Sol. $P_M^0 = 450$

$P_N^0 = 700$

$P_M = x_M\, 450$

$P_N = x_N\, 700$

$Y_M P_T = P_M$

$Y_N P_T = P_N$

$\frac{Y_M P_T}{Y_N P_T} = \frac{x_M}{x_N}\frac{450}{700}$

$\frac{Y_M}{Y_N} = \frac{x_M}{x_N}(0:64)$

$\frac{Y_M}{Y_N} > \frac{x_M}{x_N}$

2. Among the following, the set of parameters that represents path functions, is :

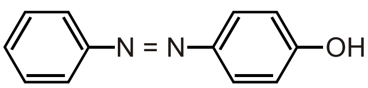
(A) q + w (B) q (C) w (D) H – TS

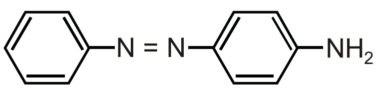
(1) (A) and (D) (2*) (B) and (C) (3) (A), (B) and (C) (4) (B), (C) and (D)

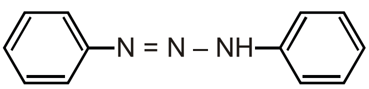
Sol. $\Delta U = q + w$

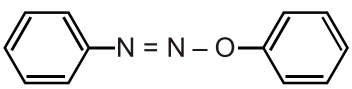
$\Delta G = \Delta H - T\Delta S$

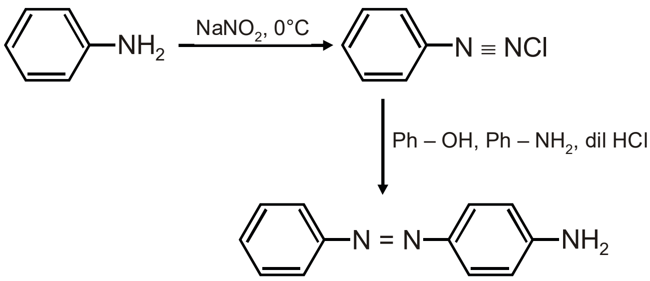
3. Aniline dissolved in dilute HCl is reacted with sodium nitrite at 0ºC. This solution was added dropwise to a solution containing equimolar mixture of aniline and phenol in dil. HCl. The structure of the major product is

(1) 

(2*) 

(3) 

(4) 

Sol. 

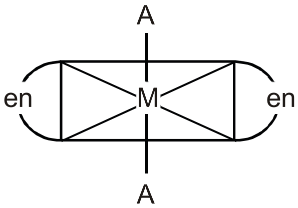
4. The number of water molecule(s) not coordinated to copper ion directly in $CuSO_4 \cdot 5H_2O$, is :

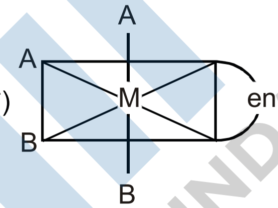
(1) 3 (2) 4 (3) 2 (4*) 1

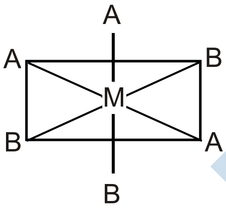
Sol. $[Cu(H_2O)_4]SO_4.H_2O$

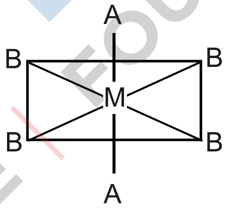
5. The one that will show optical activity is :

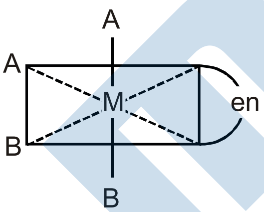
(en = ethane-1,2-diamine)

(1) 

(2*) 

(3) 

(4) 

Sol. 

No plane of symmetry

6. Excessive release of CO_2 into the atmosphere results in :

(1) polar vortex (2) formation of smog (3*) global warming (4) depletion of ozone

Sol. Fact based (Given in NCERT)

7. For any given series of spectral lines of atomic hydrogen, let $\Delta\bar{v} = \bar{v}_{max} - \bar{v}_{min}$ be the difference in maximum and minimum frequencies in cm^{-1}. The ratio $\Delta\bar{v}_{Lyman}/\Delta\bar{v}_{Balmer}$ is :

(1) 5 : 4 (2*) 9 : 4 (3) 27 : 5 (4) 4 : 1

Sol. $\Delta\bar{V}_{Lymann} = \frac{R_H}{4}$

$\Delta\bar{V}_{Balmer} = \frac{R_H}{9}$

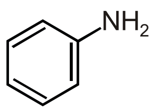
$\Rightarrow \frac{\Delta\bar{V}_{Lymann}}{\Delta\bar{V}_{Balmer}} = \frac{9}{4}$

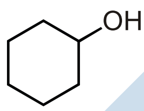
Formula

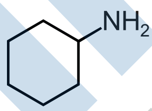
$\bar{V} = R_H\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

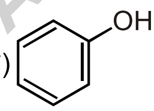
8. The organic compound that gives following qualitative analysis is :

	Test	**Inference**
(a)	Dil. HCl	Insoluble
(b)	NaOH solution	soluble
(c)	Br_2/water	Decolourization

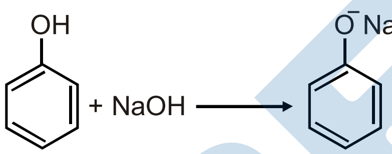
(1) 

(2) 

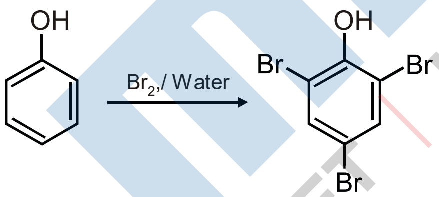
(3) 

(4*) 

Sol. Phenol being acidic insoluble in HCl

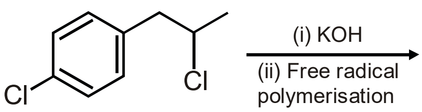


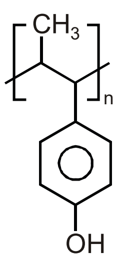
+ H_2O Soluble in NaOH

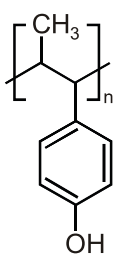


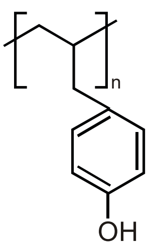
Decolourises Bromine water

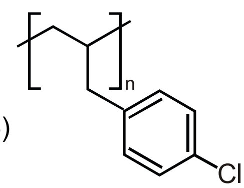
9. The major product of the following reaction is



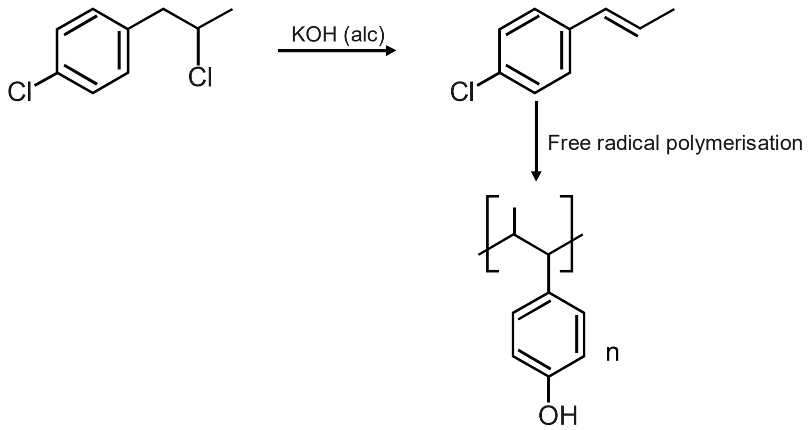
(1) 

(2*) 

(3) 

(4) 

Sol.



10. The aerosol is a kind of colloid in which :

(1) gas is dispersed in solid (2) liquid is dispersed in water

(3*) solid is dispersed in gas (4) gas is dispersed in liquid

Sol. Fact based (Given in NCERT)

11. Which of the following statements is not true about sucrose?

(1*) The glycosidic linkage is present between C_1 of α -glucose and C_1 of β -fructose

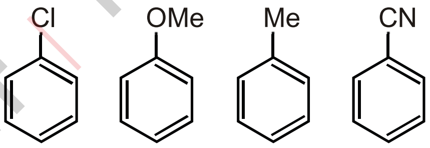
(2) It is a non reducing sugar

(3) On hydrolysis, it produces glucose and fructose

(4) It is also named as invert sugar

Sol. C_1 of α-glucose and C_2 of β-fructose

12. The increasing order of reactivity of the following compounds towards aromatic electrophilic substitution reaction is :

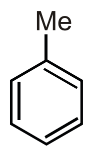


A B C D

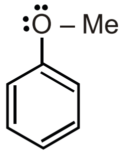
(1) A < B < C < D (2) B < C < A < D (3*) D < A < C < B (4) D < B < A < C

Sol.

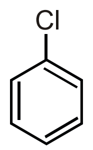
Hyperconjugation



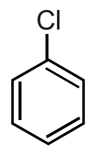
+ R effect



– R effect



+ I effect



13. For a reaction,

$N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$;

identify dihydrogen (H_2) as a limiting reagent in the following reaction mixtures.

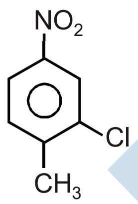
(1) 28g of N_2 + 6g of H_2 (2*) 56g of N_2 + 10g of H_2

(3) 14g of N_2 + 4g of H_2 (4) 35g of N_2 + 8g of H_2

Sol. 56 g of N_2 means 2 mole

2 mole of N_2 requires 6 mole of H_2 for complete reaction but available H_2 is 10 g i.e. 5 mole hence H_2 is limiting reagent.

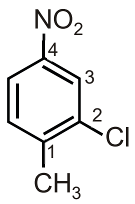
14. The correct IUPAC name of the following compound is :



(1) 5-chloro-4-methyl-1-nitrobenzene (2*) 2-chloro-1-methyl-4-nitrobenzene

(3) 2-methyl-5-nitro-1-chlorobenzene (4) 3-chloro-4-methyl-1-nitrobenzene

Sol.



15. C_{60}, an allotrope of carbon contains :

(1*) 20 hexagons and 12 pentagons. (2) 12 hexagons and 20 pentagons.

(3) 16 hexagons and 16 pentagons. (4) 18 hexagons and 14 pentagons.

Sol. Fact based (Given in NCERT)

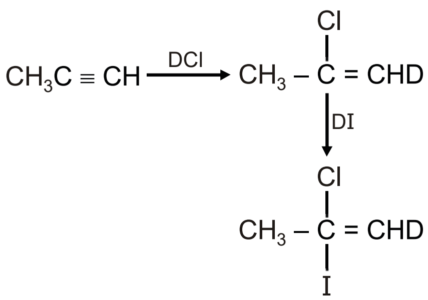
16. The major product of the following reaction is

$CH_3C \equiv CH \xrightarrow[\text{(ii) DI}]{\text{(i)DCl (1 equiv.)}}$

(1) $CH_3CD(I)CHD(Cl)$ (2*) $CH_3C(I)(Cl)CHD_2$

(3) $CH_3CD_2CH(Cl)(I)$ (4) $CH_3CD(Cl)CHD(I)$

Sol.



17. The ore that contains the metal in the form of fluoride is :

(1) sphalerite (2) malachite (3) magnetite (4*) cryolite

Sol. Cryolite is $Na_3[AlF_6]$

18. Among the following, the molecule expected to be stabilized by anion formation is :

C_2, O_2, NO, F_2

(1) NO (2) O_2 (3*) C_2 (3) F_2

Sol. C_2 configuration

$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \begin{matrix} \pi 2p_y^2 \\ \pi 2p_z^2 \end{matrix} \sigma 2Px$

C_2^- configuration

$\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \begin{matrix} \pi 2p_y^2 \\ \pi 2p_z^2 \end{matrix} \sigma 2P_x^1$

19. Consider the van der Waals constants, a and b, for the following gases.

Gas	**Ar**	**Ne**	**Kr**	**Xe**
a/ (atm dm^6 mol^{-2})	1.3	0.2	5.1	4.1
b/ (10^{-2} dm^3 mol^{-1}	3.2	1.7	1.0	5.0

Which gas is expected to have the highest critical temperature?

(1) Ne (2*) Kr (3) Xe (4) Ar

Sol. $T_C = \dfrac{8a}{27Rb}$

20. The osmotic pressure of a dilute solution of an ionic compound XY in water is four times that of a solution of 0.01 M $BaCl_2$ in water. Assuming complete dissociation of the given ionic compounds in water, the concentration of XY (in mol L^{-1}) in solution is :

(1) 4×10^{-4} (2*) 6×10^{-2} (3) 16×10^{-4} (4) 4×10^{-2}

Sol. $\pi_{xy} = 4\pi_{BaCl_2}$

π_{xy} = iCRT

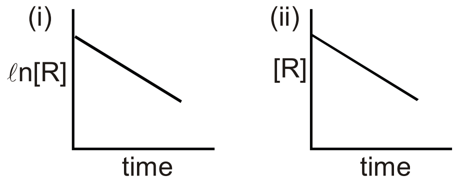
π_{xy} = 2 CRT

$\pi_{BaCl_2} = 3 \times 0.01\, RT$

RT = 12 × 0.01 RT

$\dfrac{12 \times 0.01}{2} = 0.06$

21. The given plots represent the variation of the concentration of a reactant R with time for two different reactions (i) and (ii). The respective orders of the reactions are :



(1) 0, 1 (2) 1, 1 (3) 0, 2 (4*) 1, 0

Sol. For first order reaction

$\ell n[R]_t = -Kt + \ell n[R]_o$

For zero order reaction

$[R]_t = -Kt + [R]_o$

Where 'R' is reactant.

22. The major product of the following reaction is :

$CH_3CH{=}CHCO_2CH_3 \xrightarrow{LiAlH_4}$

(1) $CH_3CH_2CH_2CH_2OH$ (2) $CH_3CH_2CH_2CHO$

(3*) $CH_3CH = CHCH_2OH$ (4) $CH_3CH_2CH_2CO_2CH_3$

Sol. $LiAlH_4$ does not reduce double bond it reduces ester in alcohol.

23. Match the catalysts (**Column I**) with products (**Column II**).

	Column I (Catalyst)		**Column II (Product)**
(A)	V_2O_5	(i)	Polyethylene
(B)	$TiCl_4/Al(Me)_3$	(ii)	ethanal
(C)	$PdCl_2$	(iii)	H_2SO_4
(D)	Iron Oxide	(iv)	NH_3

(1) (A)-(iv); (B)-(iii); (C)-(ii); (D)-(i) (2*) (A)-(iii); (B)-(i); (C)-(ii); (D)-(iv)

(3) (A)-(iii); (B)-(iv); (C)-(i); (D)-(ii) (4) (A)-(ii); (B)-(iii); (C)-(i); (D)-(iv)

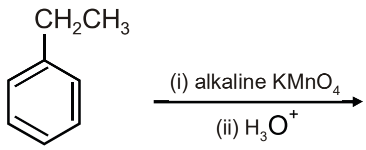
Sol. V_2O_5 is used in contact process for H_2SO_4

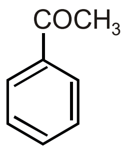
$TiCl_4/Al(Me)_3$ Ziegler natal catalyst used fin polymerization

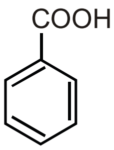
$PdCl_2$ is used in ethanol formation

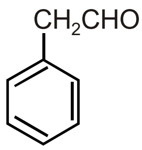
Iron oxide is used in Haber process for NH_3

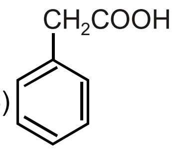
24. The major product of the following reaction is :

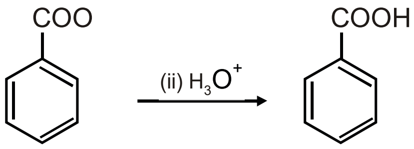


(1) 

(2*) 

(3) 

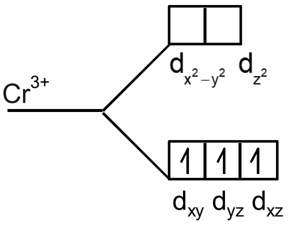
(4) 

Sol. 

25. The degenerate orbitals of $[Cr(H_2O)_6]^{3+}$ are :

(1) d_{yz} and d_{z^2} (2*) d_{xz} and d_{yz} (3) d_{z^2} and d_{xz} (4) $d_{x^2-y^2}$ and d_{xy}

Sol. Degenerate orbitals of $[Cr(H_2O)_6]^{3+}$



26. The correct order of the oxidation states of nitrogen in NO, N_2O, NO_2 and N_2O_3 is :

(1*) $N_2O < NO < N_2O_3 < NO_2$ (2) $NO_2 < N_2O_3 < NO < N_2O$

(3) $N_2O < N_2O_3 < NO < NO_2$ (4) $NO_2 < NO < N_2O_3 < N_2O$

Sol. $\overset{+2}{N}O, \overset{+1}{N_2}O, \overset{+4}{N}O_2, \overset{+3}{N_2}O_3$

27. Magnesium powder burns in air to give:

(1*) MgO and Mg_3N_2 (2) $Mg(NO_3)_2$ and Mg_3N_2

(3) MgO and $Mg(NO_3)_{20}$ (4) MgO only

Sol. $Mg + \underbrace{O_2 + N_2}_{Air} \longrightarrow MgO + Mg_2N_3$

28. The standard Gibbs energy for the given cell reaction in kJ mol^{-1} at 298 K is :

$Zn(s) + Cu^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Cu(s)$,

E° = 2 V at 298 K

(Faraday's constant, F = 96000 C mol^{-1})

(1) 384 (2) –192 (3) 192 (4*) –384

Sol. $\Delta G = -2 \times 96000 \times 2$

$\Delta G = -nFE^{o}$

$\Delta G = -2 \times 96000 \times 2$ J

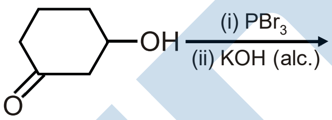
$\Delta G = -384$ kJ mol^{-1}

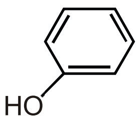
29. The element having greatest difference between its first and second ionization energies, is :

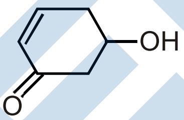
(1) Ba (2*) K (3) Ca (4) Sc

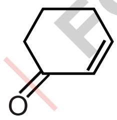
Sol. After losing one electron 'K' acquires noble gas configuration.

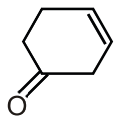
30. The major product of the following reaction is

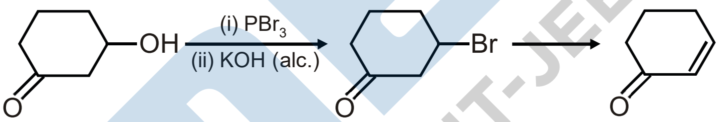


(1) 

(2) 

(3*) 

(4) 

Sol. 

PART-B-MATHEMATICS

31. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is

(1*) $\sim p \wedge \sim q$ (2) $\sim p \vee \sim q$ (3) $p \longleftrightarrow q$ (4) $p \wedge q$

Sol. $\sim (p \vee (\sim p \wedge q))$

$= \sim p \wedge \sim (\sim p \wedge q)$

$= \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$= c \vee (\sim p \wedge \sim q)$

$= (\sim p \wedge \sim q)$

32. Let $S = \left\{ \theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0 \right\}$A . Then the sum of the elements of S is

(1*) 2π (2) $\frac{5\pi}{3}$ (3) $\frac{13\pi}{6}$ (4) π

Sol. $2(1 - \sin^2\theta) + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}; \sin\theta = 2$ (reject)

roots : $\pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, -\frac{\pi}{6}, -\pi + \frac{\pi}{6}$

$\Rightarrow$ sum of values = 2π

33. All the points in the set S = $\left\{ \frac{\alpha + i}{\alpha - i} : \alpha \in R \right\} (i = \sqrt{-1})$ lie on a

(1) straight line whose slope is –1 (2) circle whose radius is $\sqrt{2}$

(3) straight line whose slope is 1 (4*) circle whose radius is 1

Sol. Let $\frac{\alpha + i}{\alpha - 1} = z$

$\Rightarrow \left| \frac{\alpha + i}{\alpha - 1} \right| = |z|$

$\Rightarrow 1 = |z|$

$\Rightarrow$ circle of radius 1

34. Let p, q $\in$ R. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then

(1) $p^2 - 4q + 12 = 0$ (2*) $q^2 + 4p + 14 = 0$ (3*) $p^2 - 4q - 12 = 0$ (4*) $q^2 - 4p - 16 = 0$

Sol. In given question p,q∈R. If we take other root as any real number α , then quadratic equation will be

$$x^2-\left(\alpha+2-\sqrt{3}\right)x+\alpha.\left(2-\sqrt{3}\right)=0$$

Now, we can have none or any of the options can be correct depending upon 'α'.

Instead of p,q∈R it should be p,q∈Q then other root will be $2+\sqrt{3}$

$\Rightarrow p=-\left(2+\sqrt{3}+2-\sqrt{3}\right)--4$ and $q=\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)=1$

$\Rightarrow p^2 - 4q - 12 = (-4)^2 - 4 = 12$

$= 16 - 16 = 0$

Option (1) is correct.

35. The value of $\int_0^{\pi/2}\frac{\sin^3 x}{\sin x+\cos x}dx$ is

(1*) $\frac{\pi-1}{4}$ (2) $\frac{\pi-2}{8}$ (3) $\frac{\pi-2}{4}$ (4) $\frac{\pi-1}{2}$

Sol. $I=\int_0^{\pi/2}\frac{\sin^3 x}{\sin x+\cos x}dx$

$$\Rightarrow I=\int_0^{\pi/4}\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}dx$$

$$=\int_0^{\pi/4}\left(1-\sin x\cos x\right)dx$$

$$=\left(x-\frac{\sin^2 x}{2}\right)_0^{\pi/4}$$

$$=\frac{\pi}{4}-\frac{1}{4}$$

$$=\frac{\pi-1}{4}$$

36. If one end of a focal chord of the parabola, $y^2 = 16x$ is at (1, 4), then the length of this focal chord is

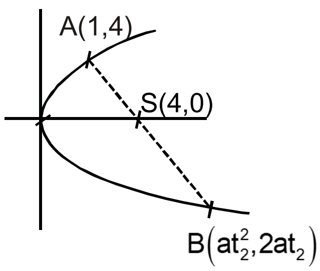
(1) 22 (2*) 25 (3) 24 (4) 20

Sol. $y^2 = 4ax = 16x \Rightarrow a = 4$

A (1,4) $\Rightarrow 2.4.t_1 = 4 \Rightarrow t_1=\frac{1}{2}$

$\therefore$ length of focal chord $=a\left(t+\frac{1}{t}\right)^2$

$$=4\left(\frac{1}{2}+2\right)^2=4.\frac{25}{4}=25$$



37. The area (in sq. units) of the region $A = \{(x, y) : x^2 \le y \le x + 2\}$ is

(1) $\frac{31}{6}$ (2) $\frac{13}{6}$ (3) $\frac{10}{3}$ (4*) $\frac{9}{2}$

Sol. $x^2 \le y \le x + 2$

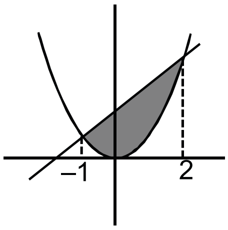
$x^2 = y;\ y = x + 2$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x - 1) = 0$

$x = 2, -1$



Area $= \int_{-1}^{2} (x+2) - x^2 dx = \frac{9}{2}$

38. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$ is

(1) $\frac{3}{2}$ (1 + cos 20°) (2) $\frac{3}{2}$ (3*) $\frac{3}{4}$ (4) $\frac{3}{4}$ + cos 20°

Sol. $\frac{1}{2}\left(2\cos^2 10° - 2\cos 10° \cos 50° + 2\cos^2 50°\right)$

$\Rightarrow \frac{1}{2}\left(1 + \cos 20° - (\cos 60° + \cos 40°) + 1 + \cos 100°\right)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20° + 2\sin 70° \sin(-30°)\right)$

$\Rightarrow \frac{1}{2}\left(\frac{3}{2} + \cos 20° - \sin 70°\right)$

$\Rightarrow \frac{3}{4}$

39. Let the sum of the first n terms of a non-constant A.P., $a_1, a_2, a_3, \ldots$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to

(1) (A, 50 + 45A) (2*) (A, 50 + 46A) (3) (50, 50 + 46A) (4) (50, 50 + 45A)

Sol. $S_n = 50n + \frac{n(n-7)}{2}A$

$T_n = S_n + S_{n-1}$

$= 50n + \frac{n(n-7)}{2}A - 50(n-1) - \frac{(n-1)(n-8)}{2}A$

$= 50n + \frac{A}{2}\left[n^2 - 7n - n^2 + 9n - 8\right]$

= 50 + A (n – 4)

d = $T_n - T_{n-1}$

= 50 + A (n – 4) – 50 – A (n – 5)

= A

T_{50} = 50 + 46A

(d, A_{50}) = (A, 50 + 46A)

40. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ $(x \neq 0)$ with y(1) = 1, is

(1) $y = \frac{4}{5}x^3 + \frac{1}{5x^2}$ (2*) $y = \frac{x^2}{4} + \frac{3}{4x^2}$ (3) $y = \frac{3}{4}x^2 + \frac{1}{4x^2}$ (4) $y = \frac{x^3}{5} + \frac{1}{5x^2}$

Sol. $x\frac{dy}{dx} + 2y = x^2 : y(1) = 1$

$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = x$ (LDE in y)

$IF = e^{\int \frac{2}{x}dx} = e^{2\ell nx} = x^2$

$y.\left(x^2\right) = \int x.x^2dx = \frac{x^4}{4} + C$

y(1) = 1

$1 = \frac{1}{4} + C \Rightarrow C = 1 - \frac{1}{4} = \frac{3}{4}$

$yx^2 = \frac{x^4}{4} + \frac{3}{4}$

$y = \frac{x^4}{4} + \frac{3}{4x^2}$

41. If the function f defined on $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ by f (x) = $\begin{cases} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} & , x \neq \frac{\pi}{4} \\ k & , x = \frac{\pi}{4} \end{cases}$ is continuous, then k is equal to

(1) $\frac{1}{\sqrt{2}}$ (2) 2 (3*) $\frac{1}{2}$ (4) 1

Sol. $\therefore$ function should be continuous at $x = \frac{\pi}{4}$

$\therefore \lim\limits_{x \to \frac{\pi}{4}} f(x) = f\left(\frac{\pi}{4}\right)$

$\Rightarrow \lim\limits_{x \to \frac{\pi}{4}} \frac{\sqrt{2}\cos x - 1}{\cot x - 1} = k$

$$\Rightarrow \lim_{x\to\frac{\pi}{4}} \frac{-\sqrt{2}\sin x}{-\operatorname{cosec}^2 x} = k \text{ (Using L Hospital Rule)}$$

$$\lim_{x\to\frac{\pi}{4}} \sqrt{2}\sin^3 x = k$$

$$\Rightarrow k = \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{2}$$

42. A plane passing through the points (0, –1, 0) and (0, 0, 1) and making an angle $\frac{\pi}{4}$ with the plane y – z + 5 = 0, also passes through the point

(1) $(-\sqrt{2}, -1, -4)$ (2) $(\sqrt{2}, -1, 4)$ (3) $(-\sqrt{2}, 1, -4)$ (4*) $(\sqrt{2}, 1, 4)$

Sol. Let ax + by + cz = 1 be the equation of the plane

$\Rightarrow$ 0 – b + 0 = 1

$\Rightarrow$ b = –1

0 + 0 + c = 1

$$\cos\theta = \left|\frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}\right|$$

$$\frac{1}{\sqrt{2}} = \frac{|0-1+1|}{\sqrt{(a^2+1+1)}\sqrt{0+1+1}}$$

$\Rightarrow$ $a^2 + 2 = 4$

$$\Rightarrow a = \pm\sqrt{2}$$

$$\Rightarrow \pm\sqrt{2} - y + z = 1$$

Now for – sign

$$\sqrt{2}, \sqrt{2} - 1 + 4 = 1$$

Hence option (4)

43. Let f (x) = 15 – | x – 10 |; x $\in$ R. Then the set of all values of x, at which the function, g(x) = f (f (x)) is not differentiable, is

(1) {10} (2) {10, 15} (3*) {5, 10, 15} (4) {5, 10, 15, 20}

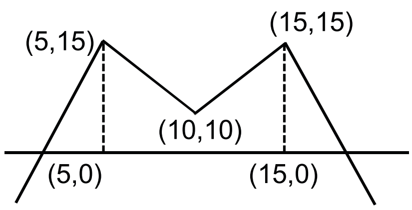
Sol. f (x) = 15 – |x – 10|, x$\in$R

f (f (x)) = 15 – |f (x) – 10|

= 15 – |15 – |x – 10| – 10|

= 15 – |5 – |x – 10||

x = 5, 10, 15 are point of non-differentiability



44. Slope of a line passing through P(2, 3) and intersecting the line, x + y = 7 at a distance of 4 units from P, is

(1) $\frac{1-\sqrt{5}}{1+\sqrt{5}}$ (2*) $\frac{1-\sqrt{7}}{1+\sqrt{7}}$ (3) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ (4) $\frac{\sqrt{5}-1}{\sqrt{5}+1}$

Sol. $x = 2 + r\cos\theta$

$y = 3 + r\sin\theta$

$\Rightarrow 2 + r\cos\theta + 3 + r\sin\theta = 7$

$\Rightarrow r(\cos\theta + \sin\theta) = 2$

$$\Rightarrow \sin\theta + \cos\theta = \frac{2}{r} = \frac{2}{\pm 4} = \pm\frac{1}{2}$$

$$\Rightarrow 1 + \sin\theta = \frac{1}{4}$$

$$\Rightarrow \sin 2\theta = -\frac{3}{4}$$

$$\Rightarrow \frac{2m}{1+m^2} = -\frac{3}{4}$$

$\Rightarrow 3m^2 + 8m + 3 = 0$

$$\Rightarrow m = \frac{-4 \pm \sqrt{7}}{1-7}$$

$$\frac{1-\sqrt{7}}{1+\sqrt{7}} = \frac{\left(1-\sqrt{7}\right)^2}{1-7} = \frac{8-2\sqrt{7}}{-6} = \frac{-4+\sqrt{7}}{3}$$

45. Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10}-1)$, where the function f satisfies f (x + y) = f (x) f (y) for all natural numbers x, y and f (1) = 2. Then the natural number 'a' is

(1) 16 (2) 4 (3) 2 (4*) 3

Sol. From the given functional equation:

$f(x) = 2^x \ \forall x \in N$

$2^{a+1} + 2^{a+2} + \dots + 2^{a+10} = 16(2^{10} - 1)$

$2^a (2 + 2^2 + \dots + 2^{10}) = 16(2^{10} - 1)$

$$2^a.\frac{2.\left(2^{10}-1\right)}{1} = 16\left(2^{10}-1\right)$$

$2^{a+1} = 16 = 2^4$

$a = 3$

46. Let α and β be the roots of the equation $x^2 + x + 1 = 0$. Then for $y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to

(1) $y^3 - 1$ (2) $y(y^2 - 3)$ (3) $y(y^2 - 1)$ (4*) y^3

Sol. Roots of the equation $x^2 + x + 1 = 0$ are $\alpha = \omega$ and $\beta = \omega^2$ where ω, ω^2 are complex cube roots of unity

$$\therefore \Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \Delta = y\begin{vmatrix} 1 & 1 & 1 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

Expanding along 1 R_1, we get

$\Delta = y.y^2 \Rightarrow D = y^3$

Or

If $\alpha = \omega^2$, $\beta = \omega$ we get same value or on expansion using $\alpha + \beta = -1$, $\alpha\beta = 1$ we get value y^3.

47. The integral $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x \, dx$ is equal to

(Here C is a constant of integration)

(1) $-3\cot^{-1/3} x + C$ (2*) $-3\tan^{-1/3} x + C$ (3) $-\frac{3}{4}\tan^{-4/3} x + C$ (4) $3\tan^{-1/3} x + C$

Sol. $$I = \int \frac{dx}{(\sin x)^{4/3}.(\cos x)^{2/3}}$$

$$I = \int \frac{dx}{\left(\frac{\sin x}{\cos x}\right)^{4/3}.\cos^2 x}$$

$$\Rightarrow I = \int \frac{\sec^2 x}{(\tan x)^{4/3}} dx$$

put $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\therefore I = \int \frac{dt}{t^{4/3}} \Rightarrow I = \frac{-3}{t^{1/3}} + c$$

$$\Rightarrow I = \frac{-3}{(\tan x)^{1/3}} + c$$

48. If the tangent to the curve, $y = x^3 + ax - b$ at the point (1, –5) is perpendicular to the line, $-x + y + 4 = 0$, then which one of the following points lies on the curve?

(1) (–2, 1) (2*) (2, –2) (3) (2, –1) (4) (–2, 2)

Sol. $y = x^3 + ax - b$

(1, – 5) lies on the curve

$\Rightarrow -5 = 1 + a - b \Rightarrow a - b = -6$ ....(i)

Also, $y' = 3x^2 + a$

$y'(1,-5) = 3 + a$ (slope of tangent)

$\because$ this tangent is $\perp$ to $-x + y + 4 = 0$

$\Rightarrow (3 + a)(1) = -1$

$\Rightarrow a = -4$ ....(ii)

By (i) and (ii) : $a = -4$, $b = 2$

$\therefore\ y = x^3 - 4x - 2$, (2, –2) lies on this curve.

49. Let S be the set of all values of x for which the tangent to the curve $y = f(x) = x^3 - x^2 - 2x$ at (x, y) is parallel to the line segment joining the points (1, f(1)) and (–1, f (–1)), then S is equal to

(1) $\left\{-\frac{1}{3}, -1\right\}$ (2*) $\left\{-\frac{1}{3}, 1\right\}$ (3) $\left\{\frac{1}{3}, 1\right\}$ (4) $\left\{\frac{1}{3}, -1\right\}$

Sol. $f(1) = 1 - 1 - 2 = -2$

$f(-1) = -1 - 1 + 2 = 0$

$$m = \frac{f(1) - f(-1)}{1 + 1} = \frac{-2 - 0}{2} = -1$$

$$\frac{dy}{dx} = x^2 - 2x - 2$$

$3x^2 - 2x - 2 = -1$

$\Rightarrow 3x^2 - 2x - 1 = 0$

$\Rightarrow (x - 1)(3x + 1) = 0$

$$\Rightarrow x = 1, -\frac{1}{3}$$

50. If the fourth term in the Binomial expansion of $\left(\frac{2}{x} + x^{\log_8 x}\right)^6$ $(x > 0)$ is 20×8^7, then a value of x is

(1) 8 (2) 8^{-2} (3*) 8^2 (4) 8^3

Sol. $$T_4 = T_{3+1} = \binom{6}{3}\left(\frac{2}{x}\right)^3 . \left(x^{\log_8 x}\right)^3$$

$$20\times 8^7 = \frac{160}{x^3}.x^{3\log_8 x}$$

$$8^6 = x^{\log_2 x} - 3$$

$$2^{18} = x^{\log_2 x - 3}$$

$\Rightarrow$ 18 = (og$_2$ x – 3) (log$_2$ x)

Let $\log_2 x = t$

$\Rightarrow t^2 - 3t - 18 = 0$

$\Rightarrow (t - 6)(t + 3) = 0$

$\Rightarrow t = 6, -3$

$\text{Log}_2\, x = 6 \Rightarrow x = 2^6 = 8^2$

$\text{Log}_2\, x = -3 \Rightarrow x = 2^{-3} = 8^{-1}$

51. If a tangent to the circle $x^2 + y^2 = 1$ intersects the coordinate axes at distinct points P and Q, then the locus of the mid-point of PQ is

(1) $x^2 + y^2 - 2xy = 0$ (2*) $x^2 + y^2 - 4x^2y^2 = 0$

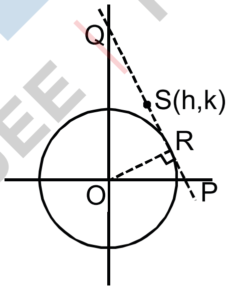
(3) $x^2 + y^2 - 2x^2y^2 = 0$ (4) $x^2 + y^2 - 16x^2y^2 = 0$

Sol. Let the mid point be S (h, k)

$\therefore$ P(2h,0) and Q(0,2k) equation of

$$PQ : \frac{x}{2h} + \frac{y}{2k} = 1$$

$\because$ PQ is tangent to circle at R (say)



$$\therefore QR = 1 \Rightarrow \left|\frac{-1}{\sqrt{\left(\frac{1}{2h}\right)^2 + \left(\frac{1}{2k}\right)^2}}\right| = 1$$

$$\Rightarrow \frac{1}{4h^2} + \frac{1}{4k^2} = 1$$

$\Rightarrow x^2 + y^2 - 4x^2y^2 = 0$

52. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to

(1) $4\sqrt{\frac{5}{3}}$ (2*) $2\sqrt{6}$ (3) $\sqrt{6}$ (4) $2\sqrt{\frac{10}{3}}$

Sol. $$S.D. = \sqrt{\frac{\sum\left(x - \bar{x}\right)^2}{n}}$$

$$\bar{x} = \frac{\sum x}{4} = \frac{-1+0+1+k}{4} = \frac{k}{4}$$

Now $\sqrt{5} = \sqrt{\frac{\left(-1-\frac{k}{4}\right)^2 + \left(0-\frac{k}{4}\right)^2 + \left(1-\frac{k}{4}\right)^2 + \left(k-\frac{k}{4}\right)^2}{4}}$

$$\Rightarrow 5 \times 4 = 2\left(1+\frac{k}{16}\right)^2 + \frac{5k^2}{8}$$

$$\Rightarrow 18 = \frac{3k^2}{4}$$

$\Rightarrow k^2 = 24$

$\Rightarrow k = 2\sqrt{6}$

53. If the function $f : R - \{1, -1\} \longrightarrow A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to

(1) $R - \{-1\}$ (2) $[0, \infty)$ (3*) $R - [-1, 0)$ (4) $R - (-1, 0)$

Sol. $y = \frac{x^2}{1-x^2}$

Range of y : $R - [-1, 0)$ for surjective function, A must be same as above range.

54. A committee of 11 members is to be formed from 8 males and 5 females. If m is the number of ways the committee is formed with at least 6 males and n is the number of ways the committee is formed with at least 3 females, then

(1) m + n = 68 (2) m = n = 68 (3*) m = n = 78 (4) n = m – 8

Sol. Since there are 8 males and 5 females. Out of these 13, if we select 11 persons, then there will be at least 6 males and at least 3 females in the selection.

$$m = n = \binom{13}{11} = \binom{13}{2} = \frac{13 \times 12}{2} = 78$$

55. If the line, $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4}$ meets the plane, $x + 2y + 3z = 15$ at a point P, then the distance of P from the origin is

(1) $\frac{7}{2}$ (2) $2\sqrt{5}$ (3*) $\frac{9}{2}$ (4) $\frac{\sqrt{5}}{2}$

Sol. Any point on the given line can be

$(1 + 2\lambda, -1 + 3\lambda, 2 + 4\lambda)$; $\lambda \in R$

Put in plane

$1 + 2\lambda + (-2 + 6\lambda) + (6 + 12\lambda) = 15$

$20\lambda + 5 = 15$

$20\lambda = 10$

$\lambda = \frac{1}{2}$

$\therefore$ Point $\left(2, \frac{1}{2}, 4\right)$

56. Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to

(1) $3\hat{i} - 9\hat{j} - 5\hat{k}$ (2) $\frac{1}{2}\left(3\hat{i} - 9\hat{j} + 5\hat{k}\right)$ (3) $-3\hat{i} + 9\hat{j} + 5\hat{k}$ (4*) $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

Sol. $\vec{\alpha} = 3\hat{i} + \hat{j}$

$\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{\beta} = \overrightarrow{\beta_1} - \overrightarrow{\beta_2}$

$\overrightarrow{\beta_1} = \lambda\left(3\hat{i} + \hat{j}\right), \overrightarrow{\beta_2} = \lambda\left(3\hat{i} + \hat{j}\right) - 2\hat{i} - \hat{j} + 3\hat{k}$

$\overrightarrow{\beta_2} . \vec{\alpha} = 0$

$(3\lambda - 2).3 + (\lambda + 1) = 0$

$9\lambda - 6 + \lambda + 1 = 0$

$\lambda = \frac{1}{2}$

$\Rightarrow \overrightarrow{\beta_1} = \frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}$

$\Rightarrow \overrightarrow{\beta_2} = -\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

Now, $\vec{\beta}_1 \times \vec{\beta}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{3}{2} & -3 \end{vmatrix}$

$= \hat{i}\left(-\frac{3}{2} - 0\right) - \hat{j}\left(-\frac{9}{2} - 0\right) + \hat{k}\left(\frac{9}{4} + \frac{1}{4}\right)$

$= -\frac{3}{2}\hat{i} + \frac{9}{2}\hat{j} + \frac{5}{2}\hat{k}$

$= \frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

57. If $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdot\ldots\ldots\ldots\cdot\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$, then the inverse of $\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$ is

(1) $\begin{bmatrix}1 & -12\\0 & 1\end{bmatrix}$ (2*) $\begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$ (3) $\begin{bmatrix}1 & 0\\12 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\13 & 1\end{bmatrix}$

Sol. $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\cdot\ldots\ldots\ldots\cdot\begin{bmatrix}1 & n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 1+2+3+\ldots\ldots+n-1\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & 78\\0 & 1\end{bmatrix}$

$\Rightarrow \frac{n(n-1)}{2}=78 \Rightarrow n=13,-12(\text{reject})$

$\therefore$ we have to find inverse of $\begin{bmatrix}1 & 13\\0 & 1\end{bmatrix}$

$\therefore \begin{bmatrix}1 & -13\\0 & 1\end{bmatrix}$

58. If f (x) is a non-zero polynomial of degree four, having local extreme points at x = –1, 0, 1; then the set S = {x $\in$ R : f (x) = f (0)} contains exactly

(1) two irrational and two rational numbers
(2) four irrational numbers
(3) four rational numbers
(4*) two irrational and one rational number

Sol. f '(x) = λ(x + 1)(x – 0)(x –1) = λ (x^3 – x)

$\Rightarrow f(x)=\lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+\mu$

Now f (x) = f (0)

$\Rightarrow \lambda\left(\frac{x^4}{4}-\frac{x^2}{2}\right)+\mu=\mu$

$\Rightarrow x=0, 0\pm\sqrt{2}$

Two irrational and one rational number

59. Four persons can hit a target correctly with probabilities $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ and $\frac{1}{8}$ respectively. If all hit at the target independently, then the probability that the target would be hit, is

(1) $\frac{1}{192}$ (2) $\frac{7}{32}$ (3) $\frac{25}{192}$ (4*) $\frac{25}{32}$

Sol. Let persons be A, B, C, D

P (Hit) = 1 – P (none of them hits)

$=1-P(\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D})$

$$=1-P(\bar{A}).P(\bar{B}).P(\bar{C}).P(\bar{D})$$

$$=1-\frac{1}{2}.\frac{2}{3}.\frac{3}{4}.\frac{7}{8}$$

$$=\frac{25}{32}$$

60. If the line y = mx + $7\sqrt{3}$ is normal to the hyperbola $\frac{x^2}{24}-\frac{y^2}{18}=1$, then a value of m is

(1) $\frac{\sqrt{15}}{2}$ (2*) $\frac{2}{\sqrt{5}}$ (3) $\frac{3}{\sqrt{5}}$ (4) $\frac{\sqrt{5}}{2}$

Sol. $\frac{x^2}{24}-\frac{y^2}{18}=1 \Rightarrow a=\sqrt{24} : b=\sqrt{18}$

Parametric normal:

$$\sqrt{24}\cos\theta.x+\sqrt{18}.y\cot\theta=42$$

At $x=0, y=\frac{42}{\sqrt{18}}\tan\theta=7\sqrt{3}$ (from given equation)

$$\Rightarrow \tan\theta=\sqrt{\frac{3}{2}} \Rightarrow \sin\theta=\pm\sqrt{\frac{3}{5}}$$

slope of parametric normal $=\frac{-\sqrt{24}\cos\theta}{\sqrt{18}\cot\theta}=m$

$$\Rightarrow m=-\sqrt{\frac{4}{3}}\sin\theta=-\sqrt{\frac{2}{5}} \text{ or } \frac{2}{\sqrt{5}}$$

PART-C-PHYSICS

61. The electric field of light wave is given as $\vec{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{x}\frac{N}{C}$. This light falls on a metal plate of work function 2eV. The stopping potential of the photo-electrons is :

Given, E (in eV) = $\frac{12375}{\lambda(\text{in Å})}$

(1) 2.48 V (2*) 0.48 V (3) 0.72 V (4) 2.0 V

Sol. $\omega = 6 \times 10^{14} \times 2\pi$

$f = 6 \times 10^{14}$

$C = f\lambda$

$\lambda = \frac{C}{f} = \frac{3\times10^8}{6\times10^{14}} = 5000$ Å

Energy of photon $\Rightarrow \frac{12375}{5000} = 2.475$ eV

From Einstein's equation

$KE_{max} = E - \phi$

$eV_s = E - \phi$

$eV_s = 2.475 - 2$

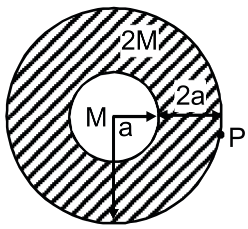
$eV_o = 0.475$ eV

$V_o = 0.48$ V

62. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be

(1) $\frac{2GM}{9a^2}$ (2) $\frac{2GM}{3a^2}$ (3*) $\frac{GM}{3a^2}$ (4) $\frac{GM}{9a^2}$

Sol. We use Gauss's Law for gravitation



$g\cdot4\pi r^2$ = (Mass enclosed) $4\pi G$

$g = \frac{3M4\pi G}{4\pi(3a)^2} = \frac{GM}{3a^2}$

63. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane. : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is :

(1) 2 : 3 : 4 (2) 4 : 3 : 2 (3*) 14 : 15 : 20 (4) 10 : 15 : 7

Sol. $\frac{1}{2}\left(m+\frac{1}{R^2}\right)v^2 = mgh$

If radius of gyration is k, then

$$h = \frac{\left(1+\frac{k^2}{R^2}\right)v^2}{2g};\ \frac{k_{ring}}{R_{ring}} = 1,\ \frac{k_{solid\ cylinder}}{R_{solid\ cylinder}} = \frac{1}{\sqrt{2}}$$

$$\frac{k_{solid\ sphere}}{R_{solid\ sphere}} = \sqrt{\frac{2}{5}}$$

$$H_1 : h_2 : h_3 :: (1+1) : \left(1+\frac{1}{2}\right) : \left(1+\frac{2}{5}\right) :: 20 : 15 : 14$$

Therefore most appropriate option is (B)

Although which is not in correct sequence

64. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?

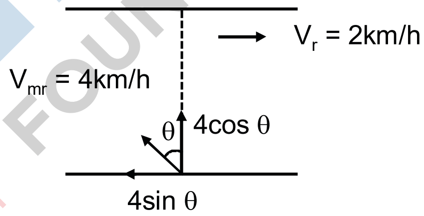
(1) 60° (2) 150° (3) 90° (4*) 120°

Sol. For swimmer to cross the river straight

$\Rightarrow$ $4 \sin \theta = 2$

$\Rightarrow$ $\sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

So, angle with direction of river flow = 90° + θ = 120°.



65. A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is :

(1) 0.42 Nm (2*) 0.27 Nm (3) 0.55 Nm (4) 0.38 Nm

Sol. $|\vec{t}| = |\vec{M} \times \vec{B}|$

τ = NI × A × B × sin 45°

τ = 0.27 Nm

66. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is :

(1) M (2) 4M (3) $\frac{M}{2}$ (4*) 2M

Sol. Height of liquid rise in capillary tube $h = \frac{2T\cos\theta_c}{\rho rg}$

$\Rightarrow \quad h \propto \frac{1}{r}$

When radius becomes double height become half

$\therefore \quad h' = \frac{h}{2}$

Now, M = $\pi r^2 h \times \rho$ and M' = $\pi(2r)^2$ (h/2) × ρ = 2M.

67. A capacitor with capacitance 5μF is charged to 5μC. If the plates are pulled apart to reduce the capacitance to 2μF, how much work is done?

(1) 2.55×10^{-6} J (2*) 3.75×10^{-6} J (3) 6.25×10^{-6} J (4) 2.16×10^{-6} J

Sol. Work done = ΔU

$$= U_f - U_i$$

$$= \frac{q^2}{2C_r} - \frac{q^2}{2C_i}$$

$$= \frac{(5\times10^{-6})^2}{2}\left(\frac{1}{2\times10^{-6}} - \frac{1}{5\times10^{-6}}\right)$$

$$= \frac{15}{4}\times10^{-6}$$

$$= 3.75 \times 10^{-6} \text{ J}$$

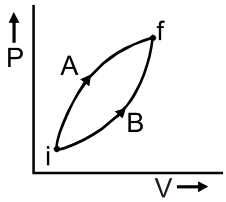
68. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is :

(1) 0.24 m (2*) 0.32 m (3) 0.16 m (4) 1.60 m

Sol. $m = \frac{f}{f-u}$

$5 = \frac{-40}{-40-u}$; u = –32 cm

69. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then :



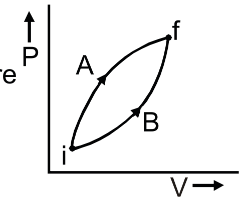
(1) $\Delta Q_A > \Delta Q_B$; $\Delta U_A > \Delta U_B$ (2) $\Delta Q_A < \Delta Q_B$; $\Delta U_A < \Delta U_B$

(3) $\Delta Q_A = \Delta Q_B$; $\Delta U_A = \Delta U_B$ (4*) $\Delta Q_A > \Delta Q_B$; $\Delta U_A = \Delta U_B$

Sol. Initial and final states for both the processes are same,

$\therefore \Delta U_A = \Delta U_B$

Work done during process A is greater than in process B. Because area is more



By First law of thermodynamics

$\Delta Q = \Delta U + W$

$\Rightarrow \Delta QA > \Delta QB$

70. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be :

(1) 642.7 nm (2) 388.9 nm (3*) 488.9 nm (4) 889.2 nm

Sol. $\frac{1}{660} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$ ...(1)

$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$ ...(B)

Divide equation (1) with (B)

$\frac{\lambda}{660} = \frac{5 \times 16}{36 \times 3}$

$\lambda = \frac{4400}{9} = 488.88 = 488.9 \text{ nm}$

71. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be :

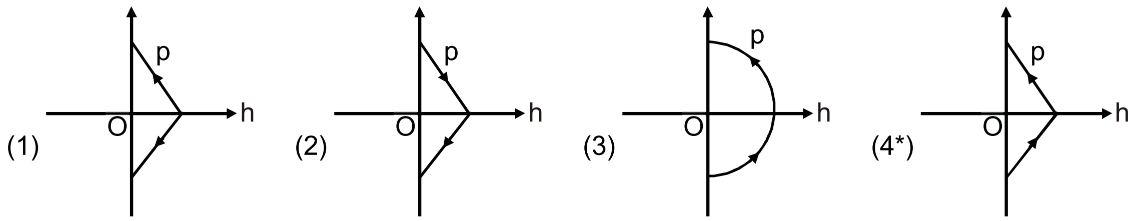
(1) $\frac{m\bar{v}^2}{7k_B}$ (2) $\frac{m\bar{v}^2}{5k_B}$ (3) $\frac{m\bar{v}^2}{6k_B}$ (4*) $\frac{m\bar{v}^2}{3k_B}$

Sol. According to equipartition energy theorem

$\frac{1}{2}m\left(v_{rms}^2\right) = 3 \times \frac{1}{2}K_bT$

$T = \frac{m\bar{v}_{rms}^2}{3k}$

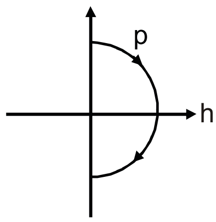
72. A ball is thrown vertically up (taken as +z-axis) from the ground. The correct momentum-height (p-h) diagram is :



Sol. Momentum p = mv ...(1)

and for motion under gravity $h = \frac{u^2 - v^2}{2g}$ ...(2)

$h = \frac{u^2 - p^2/m}{2g}$



73. An NPN transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15μA change in the base current of the amplifier. The input resistance and voltage gain are:

(1) 0.33 kΩ, 300 (2*) 0.67 kΩ, 300 (3) 0.33 kΩ, 1.5 (4) 0.67 kΩ, 200

Sol. Input current = 15×10^{-6}

Output current = 3×10^{-3}

Resistance out put = 1000

$V_{input} = 10 \times 10^{-3}$

Now $V_{input} = r_{input} \times i_{input}$

$10 \times 10^{-3} = r_{input} \times 15 \times 10^{-6}$

$r_{input} = \frac{2000}{3} = 0.67\ K\Omega.$

Voltage gain $= \frac{V_{output}}{V_{input}} = \frac{1000 \times 3 \times 10^{-3}}{10 \times 10^{-3}} = 300$

74. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to :

(1) L^2 (2) $1/L^2$ (3*) 1/L (4) L

Sol. $\phi = NBA = LI$

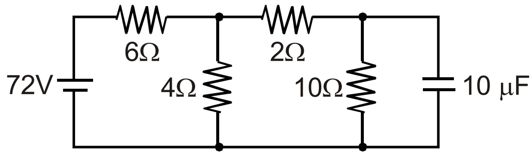
$N \mu_0 nI\pi R^2 = LI$

$N\mu_0 \frac{N}{\ell} I\pi R^2 = LI$

N and R constant

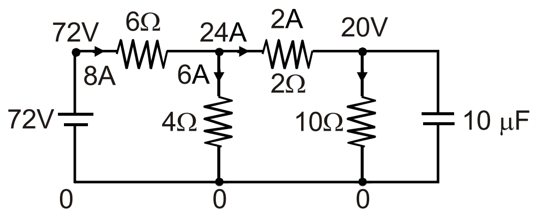
Self inductance $(L)0 \propto \frac{1}{\ell} \propto \frac{1}{\text{length}}$

75. Determine the charge on the capacitor in the following circuit :



(1*) 200μC (2) 2μC (3) 10μC (4) 60μC

Sol. Different potential is shown at different points.



q = eV

q = 10μF × 20 = 200 μC

76. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be :

(1) $80\sqrt{5}$ m/s (2) 80 m/s (3*) $100\sqrt{5}$ m/s (4) 100 m/s

Sol. $V_{rms} = \sqrt{\frac{3RT}{M_w}}$

$\Rightarrow V_{rms} \propto \sqrt{T}$

Now, $\frac{v}{200} = \sqrt{\frac{500}{400}} \Rightarrow \frac{v}{200} = \frac{\sqrt{5}}{2}$

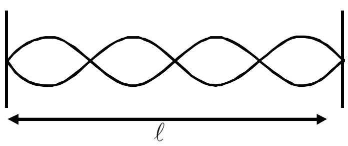
$\Rightarrow v = 100\sqrt{5}$ m/s

77. A string is clamped at both the ends and it is vibrating in its 4^{th} harmonic. The equation of the stationary wave is Y = 0.3 sin(0.157x) cos(200πt). The length of the string is : (All quantities are in SI units.)

(1) 20 m (2*) 80 m (3) 40 m (4) 60 m

Sol. 4^{th} harmonic

$4\frac{\lambda}{2} = \ell$; $2\lambda = \ell$



From equation $\frac{2\pi}{\lambda} = 0.157$

λ = 40 ; ℓ = 2λ = 80 m

78. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is :

(1) $\frac{k}{2I}\theta$ (2*) $\frac{2k}{I}\theta$ (3) $\frac{k}{4I}\theta$ (4) $\frac{k}{I}\theta$

Sol. Kinetic energy $KE = \frac{1}{2}I\omega^2 = k\theta^2$

$\Rightarrow \omega^2 = \frac{2k\theta^2}{I}$ $\Rightarrow \omega = \sqrt{\frac{2k}{I}}\theta$ ...(A)

Differentiate (A) wrt time $\rightarrow$

$$\frac{d\omega}{dt} = \alpha = \sqrt{\frac{2k}{I}}\left(\frac{d\theta}{dt}\right)$$

$$\Rightarrow \qquad \alpha = \sqrt{\frac{2k}{I}} \cdot \sqrt{\frac{2k}{I}}\,\theta\{\text{by(1)}\}$$

$$\Rightarrow \qquad \alpha = \frac{2k}{I}\theta$$

79. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(1) $2T\sqrt{\frac{1}{10}}$ (2) $4T\sqrt{\frac{1}{14}}$ (3*) $4T\sqrt{\frac{1}{15}}$ (4) $2T\sqrt{\frac{1}{14}}$

Sol. For a simple pendulum $T = 2\pi\sqrt{\frac{L}{g_{err}}}$

Situation 1: when pendulum is in air $\rightarrow g_{eff} = g$

Situation 2: when pendulum is in liquid

$$\rightarrow g_{eff} = g\left(1 - \frac{\rho_{liquid}}{\rho_{body}}\right) = g\left(1 - \frac{1}{16}\right) = \frac{15g}{16}$$

$$\text{So, } \frac{T'}{T} = \frac{2\pi\sqrt{\frac{L}{15g/16}}}{2\pi\sqrt{\frac{L}{g}}}$$

$$\Rightarrow T' = \frac{4T}{\sqrt{15}}$$

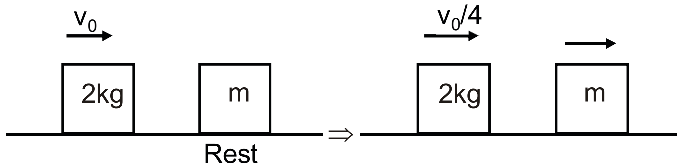
80. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body?

(1) 1.5 kg (2) 1.0 kg (3*) 1.2 kg (4) 1.8 kg

Sol. By conservation of linear momentum:

$$2v_0 = 2\left(\frac{v_0}{4}\right) + mv \Rightarrow 2v_0 = \frac{v_0}{2} + mv2$$

$$\Rightarrow \frac{3v_0}{2} = mv \qquad \ldots(1)$$

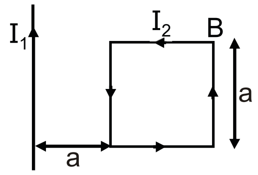


Since collision is elastic $\rightarrow$

$V_{separation} = v_{approch}$

$$\Rightarrow v - \frac{v_0}{4} = v_0 \Rightarrow m = \frac{6}{5} = 1.2 \text{ kg}$$

81. A rigid square loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to wire will be :

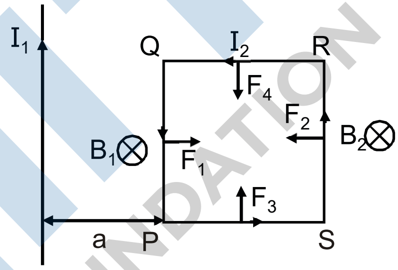


(1) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$ (2*) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

(3) Attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$ (4) zero

Sol. F_3 & F_4 cancel each other.

Force on PQ will be $F_1 = 2B_1$ a



$$= I_2 \frac{\mu_0 I_1}{2\pi a} a$$

$$= \frac{\mu_0 I_1}{2\pi a} a = \frac{\mu_0 I_1 I_2}{2\pi}$$

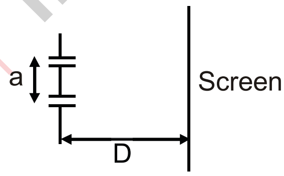
Force on RS will be $F_2 = I_2 B_2$ a

$$= I_2 \frac{\mu_0 I_1}{2\pi 2a} a$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}$$

Net force = $F_1 - F_2 = \frac{\mu_0 I_1 I_2}{4\pi}$ repulsion

82. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gest shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be



(1) $\frac{2D\lambda}{a(\mu-1)}$ (2*) $\frac{nD\lambda}{a(\mu-1)}$ (3) $\frac{2nD\lambda}{a(\mu-1)}$ (4) $\frac{D\lambda}{a(\mu-1)}$

Sol. Path difference at central maxima $\Delta x = (\mu - 1)t$, whole pattern will shift by same amount which will be given by

$(\mu-1)t\frac{D}{d} = n\frac{\lambda D}{d}$, according to eh question $t = \frac{n\lambda}{(\mu-1)}$

No option is matching, therefore question should be award bonus.

$\therefore$ Correct option should be (Bonus)

83. A moving coil galvanometer has resistance 50Ω and it indicates full deflection at 4mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to :

(1) 10 V (2) 15 V (3*) 20 V (4) 40 V

Sol. G = 50 Ω

S = 5000 Ω

$I_g = 4 \times 10^{-3}$

$V = i_g (G + S)$

$V = 4 \times 10^{-3} (50 + 5000)$

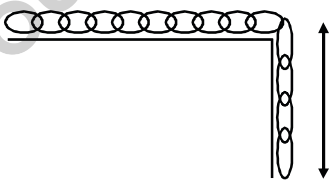
$= 4 \times 10^{-3} (5050) = 20.2$ volt

84. A uniform cable of mass 'M' and length 'L' is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)^{th}$ part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be

(1) $\frac{MgL}{n^2}$ (2) nMgL (3) $\frac{2MgL}{n^2}$ (4*) $\frac{MgL}{2n^2}$

Sol. Mass of the hanging part $= \frac{M}{n}$

$h_{COM} = \frac{L}{2n}$



Work done $W = mgh_{COM} = \left(\frac{M}{n}\right)g\left(\frac{L}{2n}\right) = \frac{MgL}{2n^2}$

85. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is :

(1*) 0.31 kg/m³ (2) 0.07 kg/m³ (3) 0.01 kg/m³ (4) 0.10 kg/m³

Sol. $\rho = \frac{m}{v}$

Maximum % error in ρ will be given by

$$\frac{\Delta p}{p} \times 100\% = \left(\frac{\Delta m}{m}\right) \times 100\% + 3\left(\frac{\Delta L}{L}\right) \times 100\% \quad \ldots(i)$$

This is not applicable as error is big.

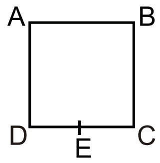
$$\rho_{min} = \frac{m_{min}}{v_{max}} = \frac{9.9}{(0.11)^3} = 7438 \text{ kg/m}^3$$

& & $\rho_{max} = \frac{m_{max}}{v_{min}} = \frac{10.1}{(0.09)^3} = 13854.6 \text{ kg/m}^3$

$\Delta p = 6416.6$ kg/m³

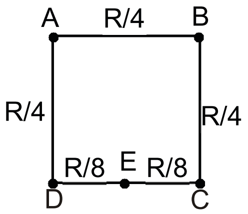
No option is matching. Therefore this question should be awarded bonus.

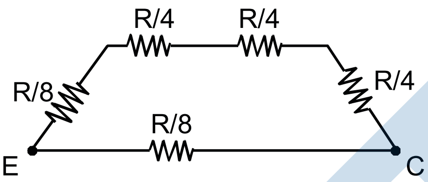
86. A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is : (E is mid-point of arm CD)



(1) $\frac{1}{16}R$ (2) R (3*) $\frac{7}{64}R$ (4) $\frac{3}{4}R$

Sol.





$$\frac{1}{R_{eq}} = \frac{8}{7R} + \frac{8}{R}$$

$$\frac{1}{R_{eq}} = \frac{8+56}{7R};\ R_{eql} = \frac{7R}{64}$$

87. The pressure wave, P = 0.01 sin [1000t – 3x] Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day, when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of T is :

(1) 11°C (2*) 4°C (3) 15°C (4) 12°C

Sol. Speed of wave from wave equation

$$v = -\frac{(\text{coddfecient of } t)}{(\text{coeffecient of } x)}$$

$$v = -\frac{1000}{(-3)} = \frac{1000}{3}$$

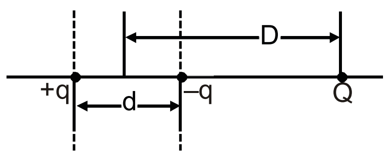
Since speed of wave $\propto \sqrt{T}$

$$\text{So } = \frac{1000}{3} = \sqrt{\frac{273}{T}}$$

$\Rightarrow$ T = 277.41 K

T = 4.41°C

88. A system of three charges are placed as shown in the figure:



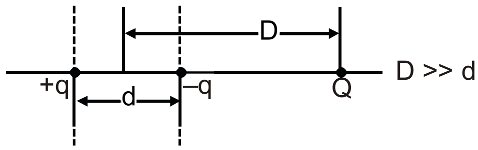
If D >> d, the potential energy of the system is best given by :

(1) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d}+\frac{qQd}{D^2}\right]$ (2*) $\frac{1}{4\pi\varepsilon_0}\left[\frac{-q^2}{d}-\frac{qQd}{D^2}\right]$

(3) $\frac{1}{4\pi\varepsilon_0}\left[\frac{-q^2}{d}-\frac{qQd}{2D^2}\right]$ (4) $\frac{1}{4\pi\varepsilon_0}\left[\frac{-q^2}{d}-\frac{2qQd}{D^2}\right]$

Sol. $U_{total} = U_{self\ of\ dipole} + U_{interaction}$

$$= -\frac{kq^2}{d}-\left(\frac{kQ}{D^2}\right)qd$$



$$= -k\left[\frac{q^2}{d}+\frac{qQd}{D^2}\right]$$

89. The magnetic field of a plane electromagnetic wave is given by :

$\vec{B}=B_0\hat{i}[\cos(kz-\omega t)]+B_1\hat{j}\cos(kz+\omega t)$ where $B_0 = 3 \times 10^{-5}$ T and $B_1 = 2 \times 10^{-6}$ T.

The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at z = 0 is closest to :

(1) 0.1 N (2*) 0.6 N (3) 0.9 N (4) 3×10^{-2} N

Sol. Maximum electric field E = (B) (C)

$$\vec{E}_0 = (3\times10^{-5})c(-\hat{j})$$

$$\vec{E}_0 = (2\times10^{-6})c(-\hat{i})$$

Maximum force

$$\vec{F}_{net} = 10^{-4}\times3\times10^{8}\sqrt{(3\times10^{-5})^2+(2\times10^{-6})^2} = 0.9\text{ N}$$

$$\vec{F}_{rms} = \frac{F_0}{\sqrt{2}} = 0.6\text{ N (approx.)}$$

90. A signal A cosωt is transmitted using ν_0 sin ω_0t as carrier wave. The correct amplitude modulated (AM) signal is :

(1*) $\nu_0\sin\omega_0 t+\frac{A}{2}\sin(\omega_0-\omega)t+\frac{A}{2}\sin(\omega_0+\omega)t$ (2) ν_0 sinω_0 t + A cosωt

(3) ν_0 sin[ω_0 (1+ 0.01 A sin ωt)t] (4) (ν_0 + A)cosωt sin ω_0t

Sol.